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Structural change in demand for fluid milk, evaporated milk, nonfat dry milk, and dry whole milk

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CHAPTER I. INTRODUCTION

Demand and supply are the fundamental factors in the price determination process. In order to do empirical price analysis, it is important to have accurate estimates and a clear understanding of past relationships between quantity and prices. In general, economists use regression analysis to get estimated demand and supply equations. This estimation technique usually assumes that the parameters of the equation are constant over all the observations. Sometimes, however, exogenous shocks in the economy, such as shifts in consumer preferences, adoption of new technology, public policy changes or changing phases of the business cycle, lead to permanent changes in behavioral relationships. Hence, it is often suggested that assuming constant parameters in a regression relationship may not be valid. It may be more plausible to assume that the parameters of the demand or supply relations are varying continuously over time, or, alternatively, change between successive time intervals. Such a change in the regression parameters is called structural change. Poirier's (1976) interpretation of structural change is that structural change occurs whenever the parameters of an economic model change within the sample period in response to forces within or outside the model.

In this context, demand and/or supply equations are envisioned as changing over time so that the constant coefficients of regression equations will not accurately represent the relationships which prevailed in any of the subperiods. In order to model structural change, we can allow the parameters of an econometric model to change so that the model provides a better approximation of the behavioral relationships.

Objectives

The purpose of this study is to apply statistical techniques to empirically examine structural change in economic relationships for milk products using a linear econometric model. If a disruptive exogenous shock affects the behavioral relationships, for example a war, then a test for structural change is easily accomplished. However, if an approximation of the economic relationship must be constructed and estimated using regression analysis, then several statistical techniques are available to estimate and test the significance of the structural change. In this study, we construct an empirical model of demand for four milk products. Since perfect knowledge about the factors that affect the demand for these products is unavailable, a statistical approach will be described and used to analyze the structural change in these retail demands.

In brief, we apply statistical techniques to empirically demonstrate the detection of structural change in the demand for four dairy products. The following procedure is used:

1. Construct empirical linear models of demand for fluid milk, evaporated milk, nonfat dry milk, and dry whole milk.
2. Estimate the point(s) at which structural change(s) appear(s) to have occurred.

3. Test whether the structural change(s) that have taken place are statistically significant.
4. Evaluate the *ex post* predictive ability of the estimated equation to provide further evidence of structural change.

Outline of Remaining Chapters

In this study, Chapter II reviews statistical techniques used to detect structural change and reviews previous empirical studies of structural change. Chapter III outlines a demand system for four dairy products which will be used in this study, describes the data which are required in this study, and discusses in detail the procedures of empirical analysis. Chapter IV presents the results of the statistical analysis of structural change. Chapter V evaluates the accuracy of consumption forecasts which incorporate structural change. Finally, Chapter VI presents the summary and conclusions of this study.

CHAPTER II. REVIEW OF LITERATURE

In recent years, much attention has been focused on the estimation of structural change. Several relevant studies are contained in the literature. In this chapter, we will first discuss the types of structural change that are under investigation. Then, studies that estimate the points at which structural change occurred will be reviewed. Third, studies which deal with methods of testing for structural change will be presented. Fourth, studies which develop models that simultaneously detect possible points and test for the significance of structural change will be discussed. Finally, studies dealing with empirical applications of structural change models will also be presented.

The Types of Structural Change

An unstable regression relationship may be caused by different kinds of shocks. According to the type of shock, we can divide the structural change of regression into two types.

The structural changes in time series is one type. During the entire period, the exogeneous shocks such as war, public policy, or taste change may result in the change of the regression relationship. Therefore, there are different regression relationships for different time periods.

The other type is the structural change in cross-sectional series. The endogeneous variable such as the price variable of demand function may have knots so that the regression relationship is formed as the grafted function. Therefore, different regression equations are formed over different segments.

Estimation of Points of Structural Change

Usually, the points of structural change are not known precisely. Thus, we should prespecify the possible points at which the structural change occurred before we test whether there is structural change. The CUSUM test is a technique for identifying the points of structural change.

The CUSUM test is the cumulative sums test. This test is suggested by Brown _et al_. (1975). The basic regression model which they considered is

$$Y_t = X_t' \beta_t + U_t, \quad t = 1, 2, \ldots, T \tag{2.1}$$

where the subscript t is the time period, Y_t is the observation on the dependent variable, and X_t is the column vector of observations on K regressors. The column vector of parameters, β_t, is written with the subscript t to indicate that it may vary with time. In addition, the error terms, U_t, are assumed normally and independently distributed with mean zero and variance σ_t^2, $t = 1, 2, \ldots, T$. The null hypothesis is H_o: $\beta_1 = \beta_2 = \ldots = \beta_T = \bar{\beta}$, which implies that the random parameters, β_t, are the same and there is no structural change over time. The CUSUM test uses the recursive residuals, W_r, and is based on the plot of the CUSUM quantities

$$V_r = \frac{1}{\hat{\sigma}} \sum_{k+1}^{r} W_j , \quad r = k+1, \ldots, T \tag{2.2}$$

where recursive residuals, W_r, are defined to be uncorrelated with zero means, constant variances and

$$W_r = \frac{Y_r - X_r' b_{r-1}}{\hat{\sigma}(1 + X_r' [X_{r-1}' X_{r-1}]^{-1} X_r)^{\frac{1}{2}}} , \quad r = k+1, \ldots, T \tag{2.3}$$

More specifically, W_r is the standardized prediction error of Y_r when predicted from $Y_1, Y_2, \ldots, Y_{r-1}$, and σ^2 denotes the estimated standard deviation computed as $\hat{\sigma}^2 = S_T/(T-k)$, where $S_T = \sum_{j=k+1}^{T} W_j^2$. Under the null hypothesis, the sequence $V_{k+1}, \ldots, V_r$ is a sequence of approximately normal variables such that

$$E(V_r) = 0,$$

$$V(V_r) = r-k, \text{ and}$$

$$C(V_r, V_s) = \min (r,s) - k. \tag{2.4}$$

If we wish to find a line such that under H_o the probability that the sample path lies above the line at any point between t=k and t=T is constant, we can form the family of pairs of straight lines through the points $\{k \pm a\sqrt{T-k}\}$, $\{T \pm 3a\sqrt{T-k}\}$, where a is a parameter. Useful pairs of values of a and α are

$$\alpha = 0.01, \quad a = 1.143$$

$$\alpha = 0.05, \quad a = 0.948$$

$$\alpha = 0.10, \quad a = 0.850 \ . \tag{2.5}$$

If the sample path travels outside the region between the lines, then the null hypothesis is rejected.

Tests for Significant Structural Change

In this section, we assume that information about the points of structural change are known *a priori* or that the points have been prespecified. In this case, the concern is whether the structural change is significant.

The works of Chow (1960), Fisher (1970), and theory of the linear spline are three of the best known and most straightforward approaches to the evaluation of structural change.

Chow test

The Chow test compares the coefficients in two regressions. The results of Chow's (1960) paper can be summarized briefly:

> To test the equality between sets of coefficients in two linear regressions, we obtain the sum of squares of residuals

assuming the equality and the sum of squares without assuming the equality. The ratio of the difference between these two sums to the latter sum, adjusted for the corresponding degrees of freedom, will be distributed as the F ratio under the null hypothesis (no structural change).

Operationally, to test the homogeneity of the sets of coefficients in two regressions, we have the models:

$Y_1 = X_1' \beta_1 + U_1$ for the first n observations with k explanatory variables (X's)

$Y_2 = X_2' \beta_2 + U_2$ for the next m observations with the same variables (X's) (2.6)

Assuming that U_1 and U_2 have identical variance-covariance structures, $\sigma_1^2 I = \sigma_2^2 I$, the model can be rewritten as:

$$\begin{matrix} Y_1 \\ Y_2 \end{matrix} = \begin{matrix} X_1 & 0 \\ 0 & X_2 \end{matrix} \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} U_1 \\ U_2 \end{matrix} \qquad (2.7)$$

Under the null hypothesis (H_o $\beta_1 = \beta_2 = \beta$, i.e., no change in the structural parameters has taken place), the restricted model is

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \beta + \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{2.8}$$

We obtain the sum of squares of the residuals under the null hypothesis using Equation (2.8) and under the alternative hypothesis using Equation (2.7). Denote these sums of squares as SS_o and SS_a, respectively. The degrees of freedom of SS_o is m + n - k and the degrees of freedom of SS_a is m + n - 2k. If the null hypothesis is true, then the ratio

$$\frac{(SS_o - SS_a)/k}{SS_a/(m-n-2k)} \tag{2.9}$$

can be shown to follow an F distribution with k and m-n-2k degrees of freedom. This F statistic can be used to test whether the null hypothesis of a single set of parameters seems consistent with the data.

Fisher's test

Fisher's approach is a generalization of Chow's. He discusses the equality test for (1) a subset of coefficients in a single regression, (2) all regression coefficients in two regressions, and (3) a subset of coefficients in two regressions. These three problems utilize the same basic theory. The equality of the entire set of regression coefficients is a special case of that considered in the problem of equality of a subset of coefficients. To test the statistical significance of a structural change between two or more specified periods, the F test is constructed. The construction of the F statistic is the same for the Chow test.

Linear spline transformation model

The testing techniques described in Chow (1960) and Fisher (1970) are two of the more popular techniques. These techniques are useful in testing for differences in parameters in different time periods or differences in parameters for qualitatively different populations in cross-section models (e.g., responses differ according to ethnicity or sex). It is possible, however, that the cause of the structural change is a quantitative and continuous variable which affects a slope of the demand function. In this event, linear spline models are chosen to represent the structural change. Poirier (1976) has defined the linear spline as the following:

> Let the set $S = (\bar{X}_1 < \bar{X}_2 < \ldots < \bar{X}_{k-1})$ of abscissa values be referred to as a mesh and the k-1 individual points X_j ($j = 1, 2, \ldots, k-1$) as interior knots or simply knots. Then, the dependent variable Y is a linear spline S(X) over S if and only if Y is a continuous piecewise linear function in X consisting of k segments defined over the k intervals $(-\infty, \bar{X}_1)$, $(\bar{X}_1, \bar{X}_2)$, ..., $(\bar{X}_{k-1}, \infty)$, respectively. Denote $Y_1, Y_2, \ldots, Y_{k-1}$ as the ordinate values at the

interior knots $\bar{X}_1$, $\bar{X}_2$, ..., $\bar{X}_{k-1}$, and Y_o and Y_k are the ordinate values which map the two end knots, $\bar{X}_o < \bar{X}_1$ and $\bar{X}_k > \bar{X}_{k-1}$, respectively.

We can get a straight line equation for the j^{th} segment as:

$$S(X) = Y_{j-1} + \frac{Y_j - Y_{j-1}}{\bar{X}_j - \bar{X}_{j-1}} (X - \bar{X}_{j-1}) \qquad (2.10)$$

where $\bar{X}_{j-1} < X < \bar{X}_j$. Poirier then defines the K transformed variables:

$$W_1 = X$$

$$W_j = \max (X - \bar{X}_j, 0)$$

$$= \begin{cases} X - \bar{X}_{j-1}, & \text{if } X > \bar{X}_{j-1} \\ 0, & \text{if } X \leq \bar{X}_{j-1} \end{cases} \qquad (2.11)$$

for $j = 2, 3, \ldots, k$.

Therefore, for any X, the linear spline S(X) can be written as

$$S(X) = \beta_0 + \beta_1 W_1 + \beta_2 W_2 + \ldots + \beta_k W_k \qquad (2.12)$$

From this spline, the slope of the j^{th} segment is $\beta_1 + \beta_2 + \ldots + \beta_j$, $j = 1, 2, \ldots, k$. Since $\beta_j = 0$ for $j = 2, 3, \ldots, k$ implies the same slope over interval j-1 and j, we can get the t-ratio corresponding to

β_j for j = 2, 3, ..., k to test the significance of the change in slope over interval j-1 and j. If the t-value corresponding to j is significant, then structural change occurs at $\bar{X}_{j-1}$.

Detecting and Testing Structural Change

Quandt's Likelihood Radio Technique (1958, 1960) was derived to estimate the points of structural change and conduct a test for the significance of structural change. The regression model used to estimate the points of structural change is

$$Y_t = X_t \beta_1 + U_{1t}, \quad \text{for } 1 \leq t \leq n_0$$

and

$$Y_t = X_t \beta_2 + U_{2t}, \quad \text{for } n_0 < t \leq n \tag{2.13}$$

where $U_{1t} \sim N(0, \sigma_1^2)$ and $U_{2t} \sim N(0, \sigma_2^2)$ and the point of structural change occurs at $t = n_0$. If n_0 is known, then we can easily estimate a separate regression equation for each of the time periods. In practice, n_0 needs to be estimated. If the variances of the errors in both the regimes are assumed equal ($\sigma_1^2 = \sigma_2^2$), we can estimate n_0 by looking at the sum of two residual sums of squares for different values of n_0 and then choose the value of n_0 which minimizes this sum. If the error variances are unequal, i.e., $\sigma_1^2 \neq \sigma_2^2$, then we can estimate n_0 by maximizing the log-likelihood function

$$L(Y|n_0^*) = (\frac{1}{2\pi})^{n/2} \sigma_1^{-n_0^*} \sigma_2^{-(n-n_0^*)}$$

$$\exp \{-\frac{1}{2\sigma_1^2} \sum_{t=1}^{n_0^*} (Y_t - X_t' \beta_1)^2$$

$$- \frac{1}{2\sigma_2^2} \sum_{t=n_0^*+1}^{n} (Y_t - X_t' \beta_2)^2\} \tag{2.14}$$

This maximization is performed by choosing an estimator for n_0, n_0^*, which maximizes the likelihood $L(Y|n_0^*)$. For testing the hypothesis that no structural change occurred, Quandt used the likelihood ratio

$$\lambda = \frac{L(\hat{W})}{L(\hat{\Omega})} \tag{2.15}$$

where $L(\hat{\Omega})$ is the unrestricted maximum of the likelihood function over the entire parameter space Ω and $L(\hat{W})$ is the maximum of the likelihood function over the space W defined by the hypothesis. In this case, we can obtain a likelihood ratio as

$$\lambda = \frac{\hat{\sigma}_1^{n_0} \hat{\sigma}_2^{n-n_0}}{\hat{\sigma}^n} \tag{2.16}$$

where $\hat{\sigma}$ is the estimated standard deviation of the residuals from a single regression over the entire sample.

The Empirical Applications

Nyankori and Miller (1982) used quarterly data from 1965 through 1979 and linear spline functions to test hypotheses about structural changes in retail demand for beef, chicken, turkey, and pork. In the absence of information about the exact point where structural change took place, the cumulative sums of squares test was used to determine points of structural change. The results of this paper showed evidence that structural change occurred during the sample period in the beef and chicken demand functions, but not in the pork and turkey demand functions.

Chavas (1983) presented a method utilizing linear economic models to investigate structural change in economic relationships. Using an approach that assumes parameters can change randomly from one period to another, Chavas addressed structural change in the demand for poultry, pork, and beef during a 1950 through 1979 sample period. This paper concluded that no structural change occurred in the demand for pork, while structural change in the demand for poultry and beef was detected in the 1970s.

Braschler (1983) specified the retail price of pork and beef as the dependent variable. Ordinary least squares were used for parameter estimation, and the switching regression model (Madalla, 1977, pp. 390-403) was used to divide an overall time period into two periods. A dichotomy of the sample period resulted in a minimum sum of error sums of squares. The Chow test was used to test the significance of structural change between these two periods. The analysis of pork data using the regression switching procedure resulted in a division of the 1950-1982 sample period into a 1950-1969 subperiod and a 1970-1982 subperiod. When the procedure was applied to the beef data, the sample period was divided into a 1950-1970 subperiod and 1971-1982 subperiod. The results of the Chow test indicate that structural change in the demand for both pork and beef was significant.

CHAPTER III. MODEL PROCEDURES AND DATA

Of primary importance in this study is the development of a method for investigating structural change. In this chapter, the model used for the analysis will be presented, the statistical test for examining structural change will be discussed, and then the data required for this research will be described.

Theoretical Model

In this study, the per capita consumption of each commodity is expressed as a linear function of its own price, the price of other goods, and income. That is, we consider a linear demand model of the form:

$$Q_{it} = a_i + b_i \left(\frac{P_{it}}{CPI_t}\right) + c_i \left(\frac{Y_t}{CPI_t}\right) + \sum_{j=1}^{3} d_{ij} \left(\frac{P_{jt}}{CPI_t}\right) + e_{it} \qquad (3.1)$$

where

Q_{it} = per capita consumption of good i in period t,

P_{it} = price of the i^{th} commodity in period t,

P_{jt} = price of the j^{th} commodity in period t,

Y_t = per capita income in period t,

CPI_t = consumer price index in period t, and

e_{it} = a random error term.

The problem is to test whether the demand relationship for specified commodities remains stable during the period under study.

The null hypothesis is that no change in the demand structure occurred during the period of study. The alternative hypothesis is that the demand relationships changed during the sample period.

The variables included in the empirical demand analysis are:

Dependent variables

FQ = per capita consumption of fluid milk in pounds,

EQ = per capita consumption of evaporated milk in pounds,

NQ = per capita consumption of nonfat dry milk in pounds, and

WQ = per capita consumption of dry whole milk in pounds; and

Independent variables

RPF = real retail price per pound for fluid milk,

RPE = real retail price per pound for evaporated milk,

RPN = real retail price per pound for nonfat dry milk,

RPW = real retail price per pound for dry whole milk, and

RY = real per capita income.

Hence, the empirical demand model for the milk system can be stated as

$$FQ = a_1 + b_1\ RPF + c_1\ RY + d_{12}\ RPE + d_{13}\ RPN + d_{14}\ RPW + e_1 \quad (3.2)$$

$$EQ = a_2 + b_2\ RPE + c_2\ RY + d_{22}\ RPF + d_{23}\ RPN + d_{24}\ RPW + e_2 \quad (3.3)$$

$$NQ = a_3 + b_3\ RPN + c_3\ RY + d_{32}\ RPF + d_{33}\ RPE + d_{34}\ RPW + e_3 \quad (3.4)$$

$$WQ = a_4 + b_4\ RPW + c_4\ RY + d_{42}\ RPF + d_{43}\ RPE + d_{44}\ RPN + e_4 \quad (3.5)$$

so that per capita consumption of fluid milk, evaporated milk, nonfat dry milk, and dry whole milk are expressed as linear functions of the price of fluid milk, evaporated milk, nonfat dry milk, dry whole milk, and income, respectively. The error term is denoted by e.

Empirical Procedure

Regression analysis will be used to examine the stability over time of these empirical relationships. The discussion of the procedures used in the empirical analysis will focus on (1) the determination of the possible point(s) at which the structural change took place, (2) tests for changes in individual coefficients of the demand equation, and (3) tests for structural change in the overall relationships.

Identifying points of structural change

The possible points of structural change are never known *a priori*, yet need to be specified so that significance tests can be conducted. There are many statistical techniques such as the Quandt's likelihood function, the cumulative sum of recursive residuals, and the cumulative sums of squares of recursive residuals which could be used to estimate the possible points at which the structural change occurred. In this study, we use the cumulative sums of squares (CUSUMSQ) test which is suggested by Brown *et al.* (1975) to prespecify the point(s) of structural change.

The CUSUMSQ test is based on recursively computed residuals and can examine structural stability by plotting of the quantities

$$S_r = \sum_{j=k+1}^{r} W_j^2 \Big/ \sum_{j=k+1}^{T} W_j^2 , \quad r = k+1, \ldots, T \tag{3.6}$$

where k is the number of regressors, T is the number of observations, and the recursive residuals, W_r, are defined to be

$$W_r = \frac{Y_r - X_r' \, b_{r-1}}{\hat{\sigma}(1 + X_r' \, [X_{r-1}' \; X_{r-1}]^{-1} \; X_r)^{\frac{1}{2}}} \, , \quad r = k+1, \ldots, T \, . \tag{3.7}$$

These recursive residuals are defined to be uncorrelated with zero means and constant variances. More formally, W_r is the standardized prediction error of Y_r when predicted from Y_1, Y_2, ..., Y_{r-1}. Under the null hypothesis, S_r may be shown to have a beta distribution with mean $(r-k)/(T-k)$, i.e., $E(S_r) = (r-k)/(T-k)$. Given a significance level, a pair of significance lines with the general form of $(r-k)/(T-k) \; C_0$ can be drawn. These two significance lines are parallel to and symmetric about the mean value line, $E(S_r)$. The significance value, C_0, can be obtained from Durbin's Table 1 (1969, p. 4). To find the significance value, C_0, we consider two cases. If T-k is even, then the value obtained by entering the table at $n = \frac{1}{2}(T-k)-1$ and α, the significance level. Alternatively, if T-k is odd, the suggested procedure is to interpolate linearly between the value for $n = \frac{1}{2}(T-k)-3/2$ and $n = \frac{1}{2}(T-k)-1/2$. This procedure provides evidence of structural change at the point where the sample path of S_r moves outside these significance lines. This technique is designed to identify graphically the possible points of structural change.

In this study, two kinds of the CUSUMSQ test are presented. One test examines structural stability by plotting the value of CUSUMSQ (S_r)

against the ordered own price sequence, the other one considers stability of regression by plotting the value of CUSUMSQ (S_r) over time. The results of the former can denote the possible break points among all own price observations. The results of the latter can identify the possible points of structural change over time.

Testing for change in individual coefficients

A linear spline function (Poirier, 1976) can be used to detect a change in an individual coefficient. The linear demand equations can be respecified as a linear spline function by the following transformation.

First, the CUSUMSQ test, when applied over observations on prices, identifies the possible points at which the structural change may have occurred in the form of changes in the price slopes of demand functions. The values of each independent variable at these break points formed a set T such that:

$$T_P = (\bar{P}_{1i} < \bar{P}_{2i} < \dots < \bar{P}_{m-1,i}) \tag{3.8}$$

or

$$T_Y = (\bar{Y}_1 < \bar{Y}_2 < \dots < \bar{Y}_{m-1}) \tag{3.9}$$

where P_i represents the price of the i^{th} commodity, the subscript m is the number of the segments in the observation sequence, and Y is per capita income. The elements of set T can be referred to as interior knots.

We can then define the transformed variables:

$$X_{it} = P_{it}$$

$$X_{mi} = \begin{cases} P_{it} - \bar{P}_{m-1,i}, & \text{if } P_{it} > \bar{P}_{m-1,i} \\ 0, & \text{if } P_{it} \leq \bar{P}_{m-1,i} \end{cases}$$

$$\text{for } m = 2, 3, \ldots, M. \tag{3.10}$$

For the independent variable Y (per capita income), the transformed variables can be expressed as:

$$W_1 = Y_t$$

$$W_m = \begin{cases} Y_t - \bar{Y}_{m-1}, & \text{if } Y_t > \bar{Y}_{m-1} \\ 0, & \text{if } Y_t \leq \bar{Y}_{m-1} \end{cases}$$

$$\text{for } m = 2, 3, \ldots, M. \tag{3.11}$$

A linear spline for P_i, $S(P_i)$, can be written as

$$S(P_i) = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_M X_M \tag{3.12}$$

and for Y, a linear spline S(Y) can be written as

$$S(Y) = c_0 + c_1 W_1 + c_2 W_2 + \ldots + c_M W_M \tag{3.13}$$

where the coefficients b_1 and c_1 represent the slope of the spline over the first interval for price and income, respectively. The remaining coefficients, b_m, m = 2, 3, ..., M, and c_m, m = 2, 3, ..., M, represent the differences between the slopes over segments m-1 and m for price and income, respectively.

By substituting $S(P_i)$ and S(Y) into the linear demand functions, a linear spline demand function for commodity i can be expressed as

$$Q_t = a_0 + b_1 X_1 + b_2 X_2 + \ldots + b_M X_M + c_1 W_1 + c_2 W_2 + \ldots + c_M W_M + \sum_{j=1}^{3} d_j P_{jt} + e_t \; . \tag{3.14}$$

This reformulation of the demand model as a linear spline function can be done for any number of independent variables so that the P_{jt}'s can also be represented as linear spline transformations.

Standard t-tests on b_m and c_m, m = 2, 3, ..., M, can be used to test for changes in individual coefficients on price and income, respectively. These hypotheses can be stated as

$$H_o\colon b_m = 0 \text{ versus } H_a\colon b_m \neq 0 \text{ for each } b_m,\ m = 2, 3, \ldots, M$$

and

$$H_o\colon c_m = 0 \text{ versus } H_a\colon c_m \neq 0 \text{ for each } c_m,\ m = 2, 3, \ldots, M.$$

Nonrejection of the null hypothesis implies in either case that there is no discrepancy in an individual coefficient over segments m-1 and m.

This test is a straightforward statistical technique which uses a t-test to examine the change of an individual coefficient.

Tests for structural change in the overall equation

In this section, a statistical test for structural change in the whole demand equation instead of the individual coefficients will be described. It is possible that the effect of a change in an individual coefficient is offset or reinforced by changes in the coefficients on the other independent variables. Therefore, a method to examine structural change in the overall relationship is necessary. The Chow test can be used to test for the existence of multiple structural changes.

It is necessary to have information about the point of structural change before using the Chow test. This information can be obtained from the CUSUMSQ test. Suppose the results of CUSUMSQ test indicate that the sample period should be divided into two segments. Let t_1 and t_2 be the number of observations in each of the respective segments, and the sum of t_1 and t_2 is the number of observations in the sample period, T. To determine whether structural change has occurred between the first and second time interval, we can estimate separate regression equations for the first t_1 observations and the additional t_2 observations. If the structure has changed, the second segment will not have the same functional form as the first segment. We can use the Chow test to determine the statistical significance of a structural change over these two specified periods.

To employ the Chow test, we represent the demand equation for commodity i as:

Period I Model:

$$Q_t = a' + b' \left(\frac{P_i}{CPI}\right)_t' + c' \left(\frac{Y}{CPI}\right)_t' + d' \sum_{j=1}^{3} \left(\frac{P_j}{CPI}\right)_t' + e_t' \qquad (3.15)$$

and

Period II Model:

$$Q_t = a'' + b'' \left(\frac{P_i}{CPI}\right)_t'' + c'' \left(\frac{Y}{CPI}\right)_t'' + d'' \sum_{j=1}^{3} \left(\frac{P_j}{CPI}\right)_t'' + e_t'' . \qquad (3.16)$$

In our empirical analysis, there are four dependent variables and five independent variables. Therefore, the period I model has t_1 observations and six explanatory variables, and the period II model has t_2 observations and the same six explanatory variables. The stochastic error terms (e's) are assumed to follow a normal distribution with zero mean and constant variance.

To perform the test, both models are estimated and the error sums of squares are calculated. Let ESS_I and ESS_{II} denote the error sum of squares for period I and period II, respectively. The degrees of freedom for ESS_I, dfe_I, is t_1-6 and the degrees of freedom for ESS_{II}, dfe_{II}, is t_2-6. We can also calculate the error sum of squares for the unrestricted model, ESS_U, where $ESS_U = ESS_I + ESS_{II}$ with degrees of freedom $dfe_U = dfe_I + dfe_{II} = t_1 + t_2 - 12 = T - 12$.

Under the null hypothesis, we can formulate and estimate a restricted model by stacking the two periods. For a specified commodity i, the restricted equation is

$$Q_t = a + b\left(\frac{P_i}{CPI}\right)_t + c\left(\frac{Y}{CPI}\right)_t + \sum_{j=1}^{3} d_j\left(\frac{P_j}{CPI}\right)_t + e_t \ . \tag{3.17}$$

The error sum of squares of this restricted equation is denoted as ESS_R with degree of freedom dfe_R = T-6.

To test the hypothesis that all parameters are the same for both periods, construct the F statistic:

$$F(6,\ T-12) = \frac{(ESS_R - ESS_U)/(dfe_R - dfe_U)}{ESS_U/dfe_U} \tag{3.18}$$

$$= \frac{(ESS_R - ESS_U)/6}{ESS_U/(T-12)} \ .$$

If the value of F ratio is not significant, then we accept the null hypothesis that there is no structural change and we can construct a single model for the entire period of analysis.

Data

Fluid milk, evaporated milk, nonfat dry milk, and dry whole milk are four dairy products which are to be investigated for structural change. The data required to study structural change are (1) per capita consumption, (2) retail prices, (3) per capita disposable incomes, and (4) the consumer price index. Per capita consumption data are available in Food Consumption, Prices, and Expenditures published by the U.S. Department of Agriculture. Price data were obtained from several issues of Agricultural Statistics. Per capita income data were obtained from the Survey of Current Business published by the U.S. Department of Commerce. Consumer price index data are obtained from various issues of Statistical Abstract of the U.S., U.S. Department of Commerce. Monetary measurements are based on 1967 prices. The time period covered for this study is 1940 to 1981, inclusive. This data, from 1940 to 1979, included 40 sets of observations and were used to estimate the demand equations for the dairy products involved. The 1980 and 1981 observations are used to judge the forecasting accuracy of the estimated demand equations. All prices and consumption data are on an annual basis at the retail level.

CHAPTER IV. RESULTS

In this chapter, the results of the CUSUMSQ test, the linear spline regression model, and the Chow test, all applied to milk demand functions, will be presented. A comparison of the estimated coefficients for different time intervals out of the sample period will also be discussed.

Results of the CUSUMSQ Test

The CUSUMSQ test is used to estimate possible points of structural change. The results of this test applied to the dairy product data are presented in Figures 1 through 8, where the cumulative sums of squared residuals (S_r), the mean value line (solid line) showing $E(S_r) = (r-k)/(T-k)$, and the significance lines (dotted lines) are all plotted. In this study, k=6, T=40, and the significance level α is given as 0.1. To construct the significance value line, the value of n should first be computed. In this study, the value of T-k is 34, an even number so that n is computed as $\frac{1}{2}(T-k) - 1$ or 16. From the Table of Significance Values for the CUSUMSQ Test (Durbin, 1969, p. 4), we can obtain the significance value $C_0 = 0.25439$. The computed values of the cumulative sums of squares (S_r) against the own price observations for fluid milk, evaporated milk, nonfat dry milk, and dry whole milk are presented in Table 1 and plotted in Figures 1, 2, 3, and 4, respectively. The computed values of the cumulative sums of squares (S_r) against sample period for these four kinds of dairy products are presented in Table 2 and plotted in Figures 5, 6, 7, and

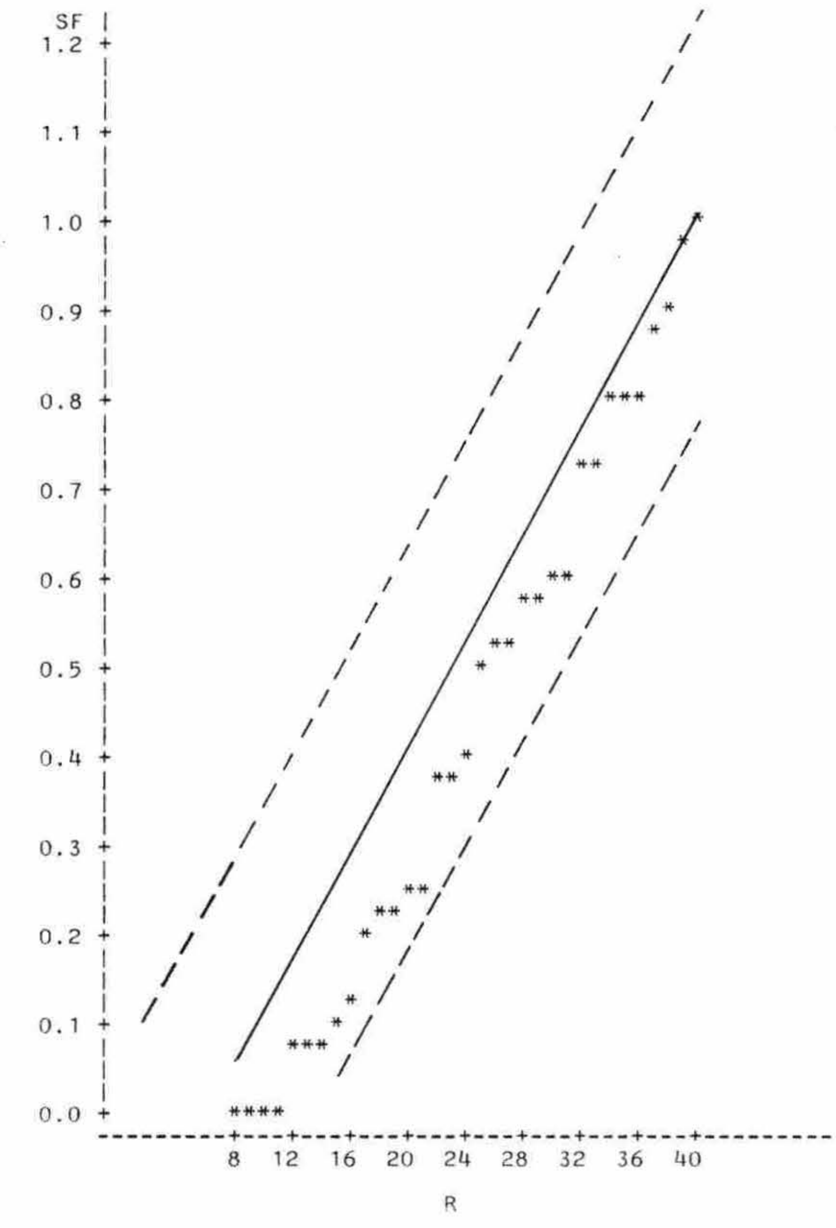

Figure 1. CUSUMSQ plot against own price observations - fluid milk

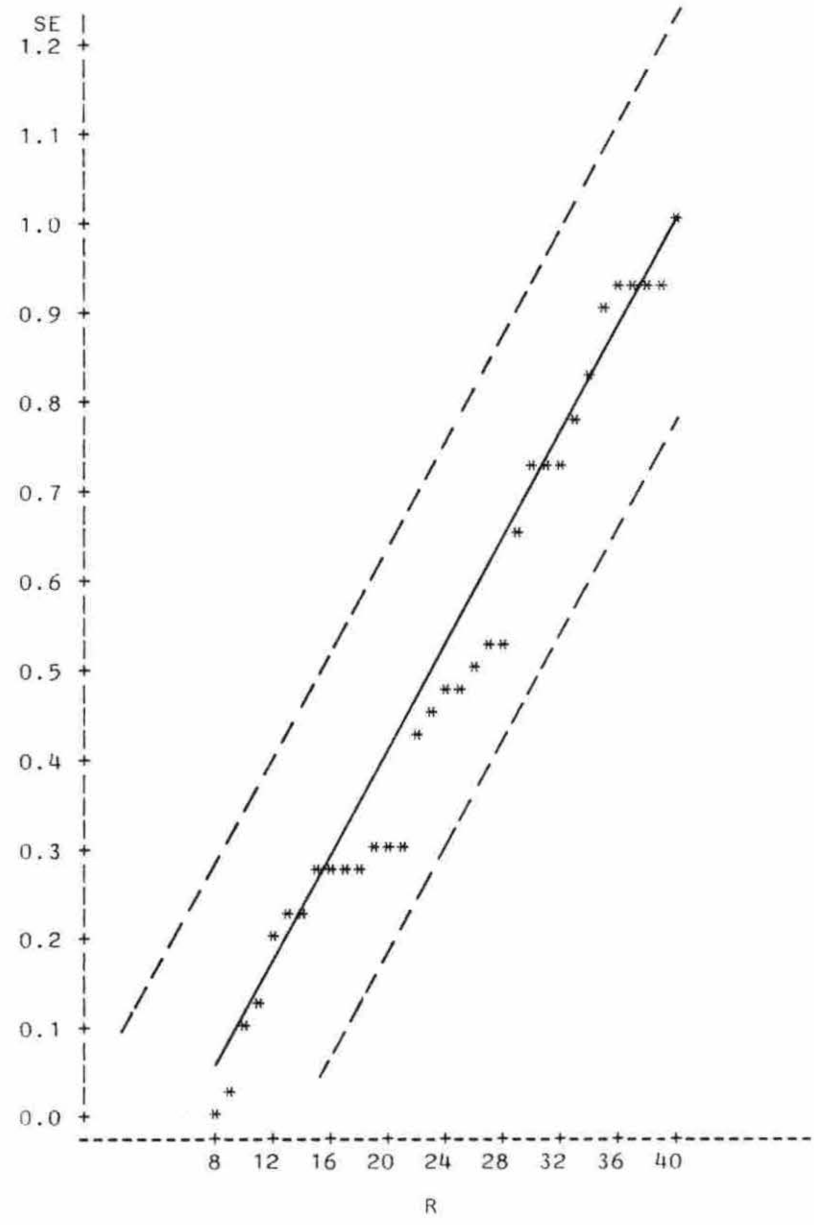

Figure 2. CUSUMSQ plot against own price observations - evaporated milk

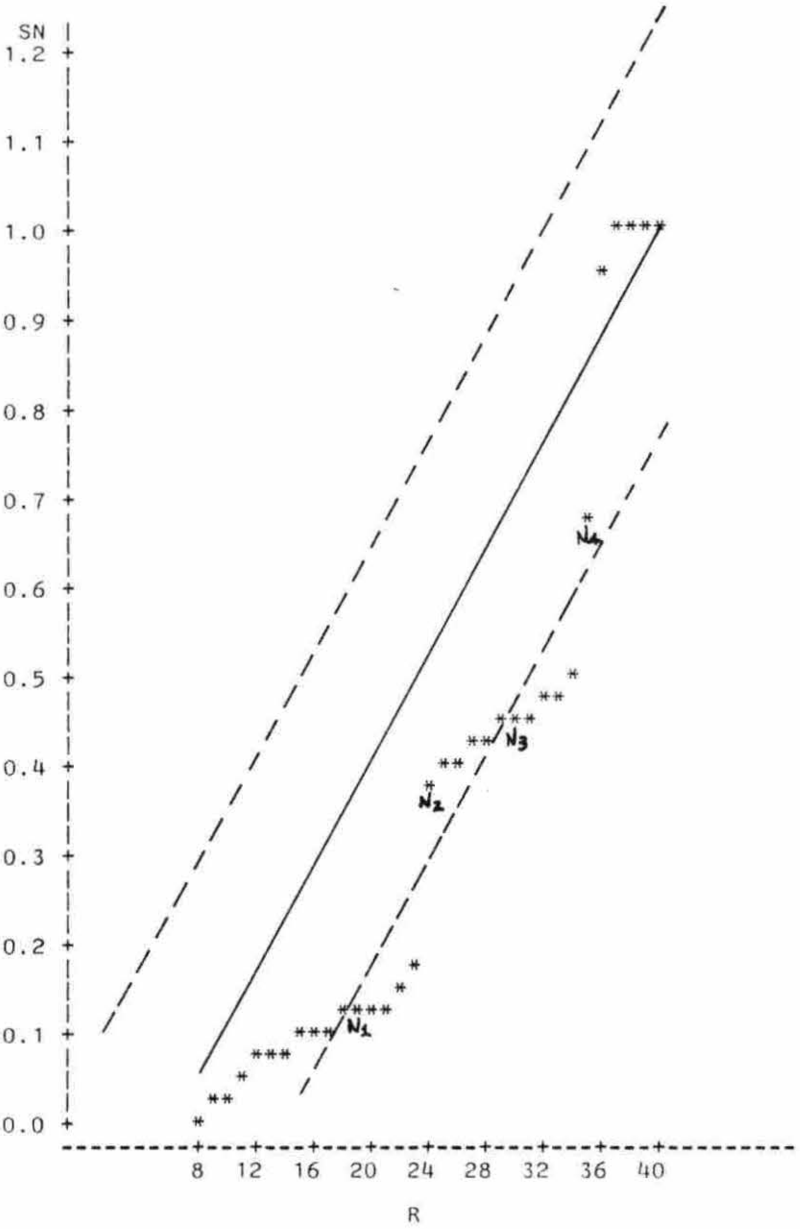

Figure 3. CUSUMSQ plot against own price observations - nonfat dry milk

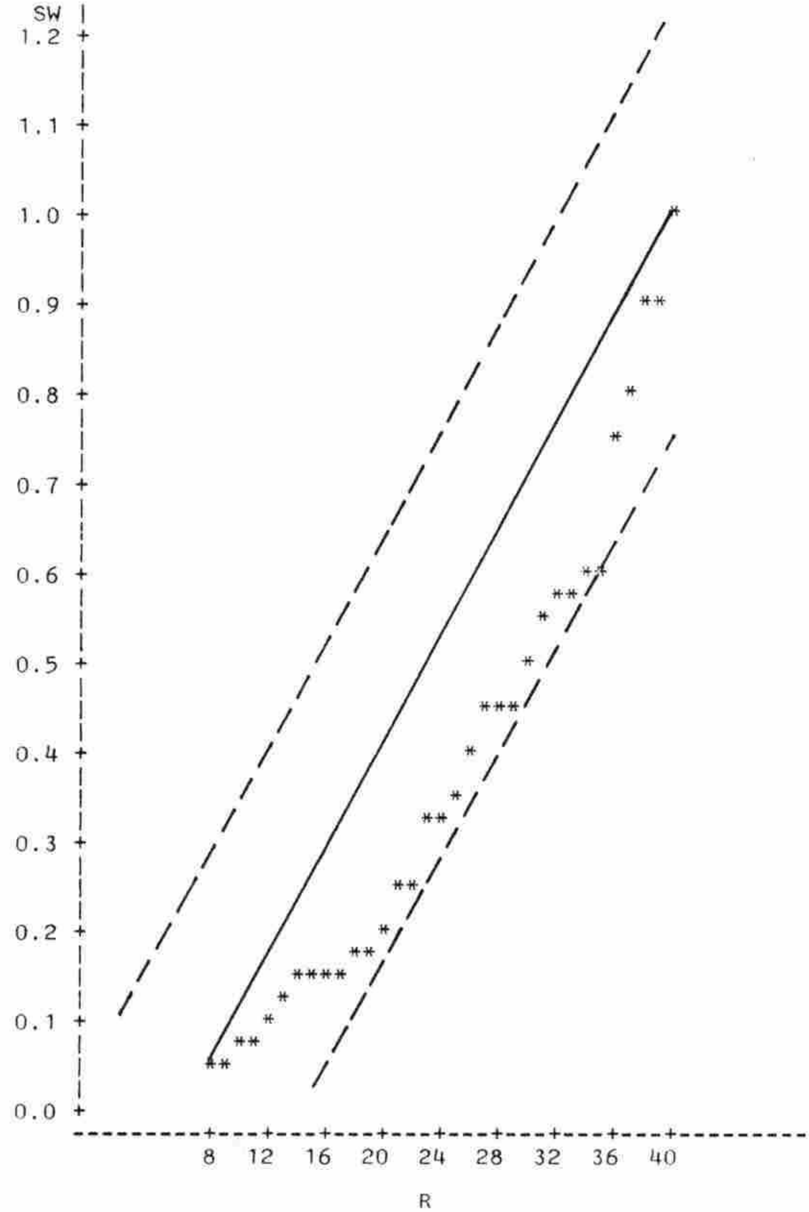

Figure 4. CUSUMSQ plot against own price
observations - dry whole milk

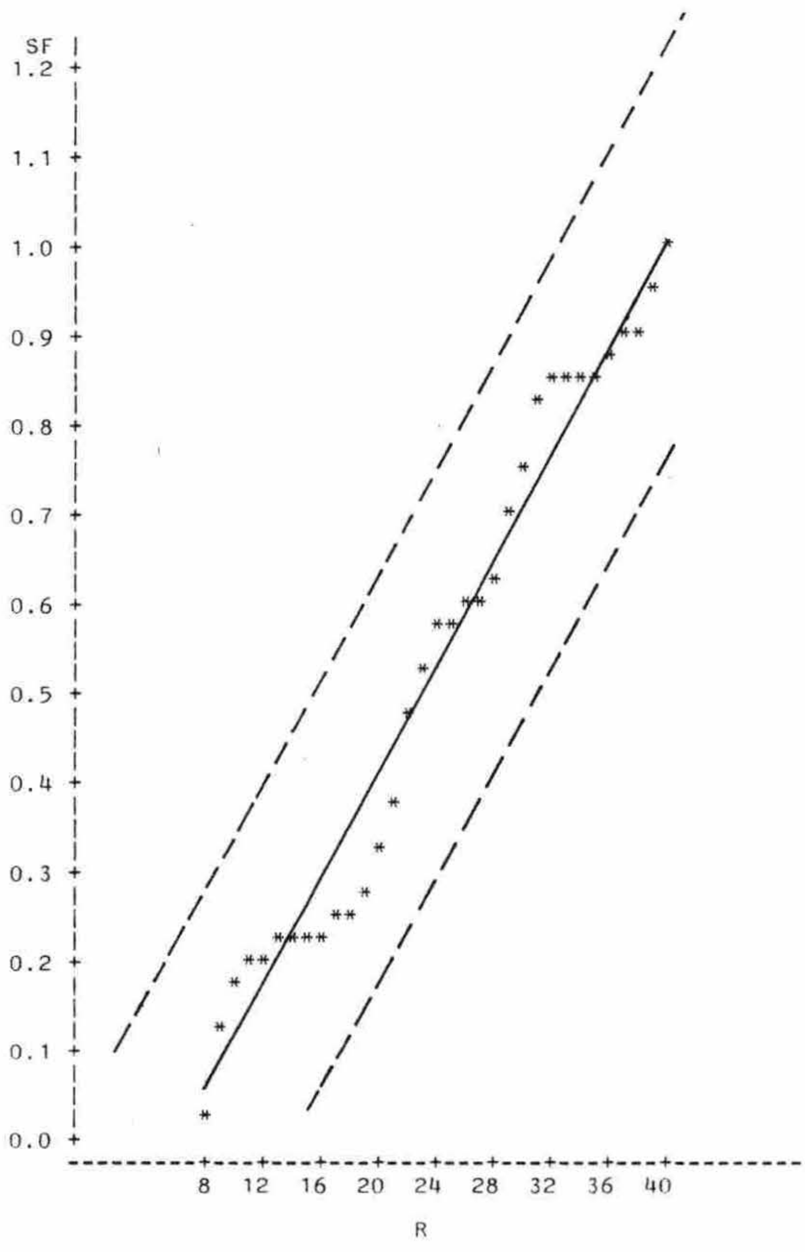

Figure 5. CUSUMSQ plot over time - fluid milk

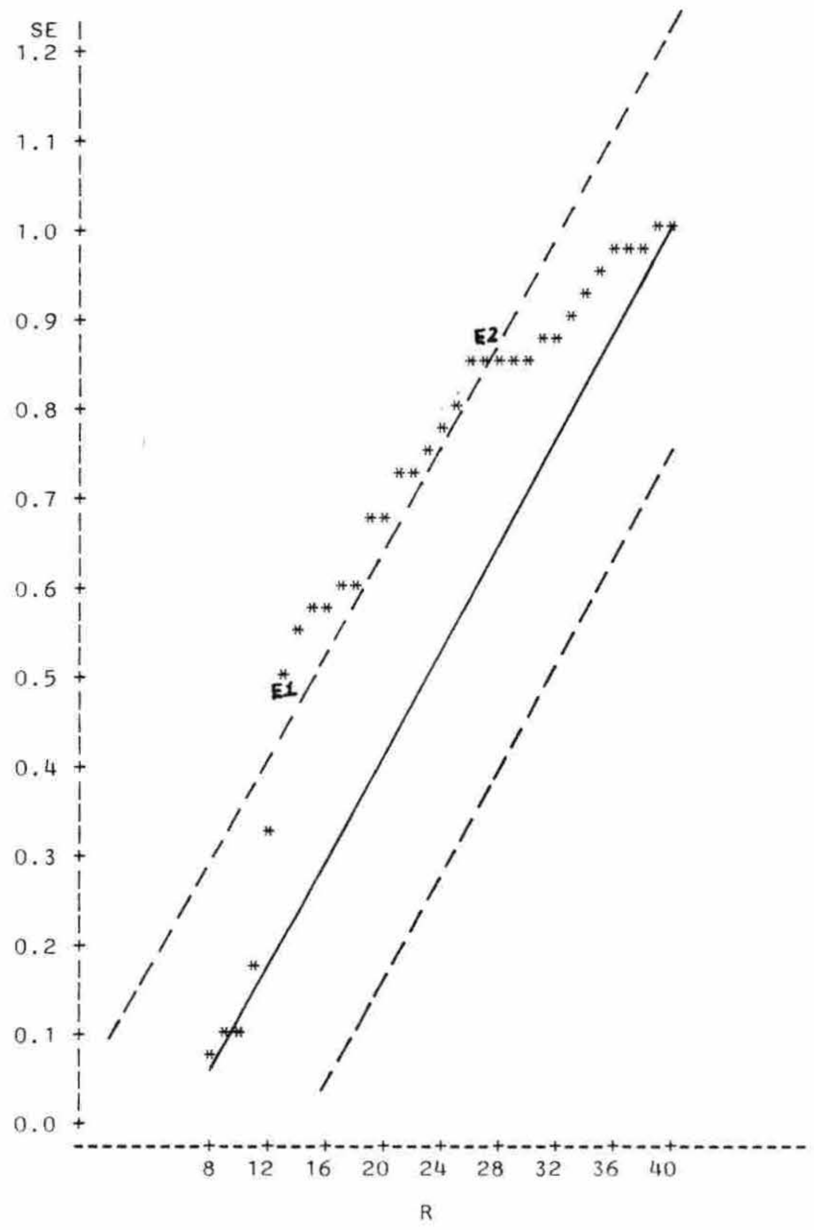

Figure 6. CUSUMSQ plot over time - evaporated milk

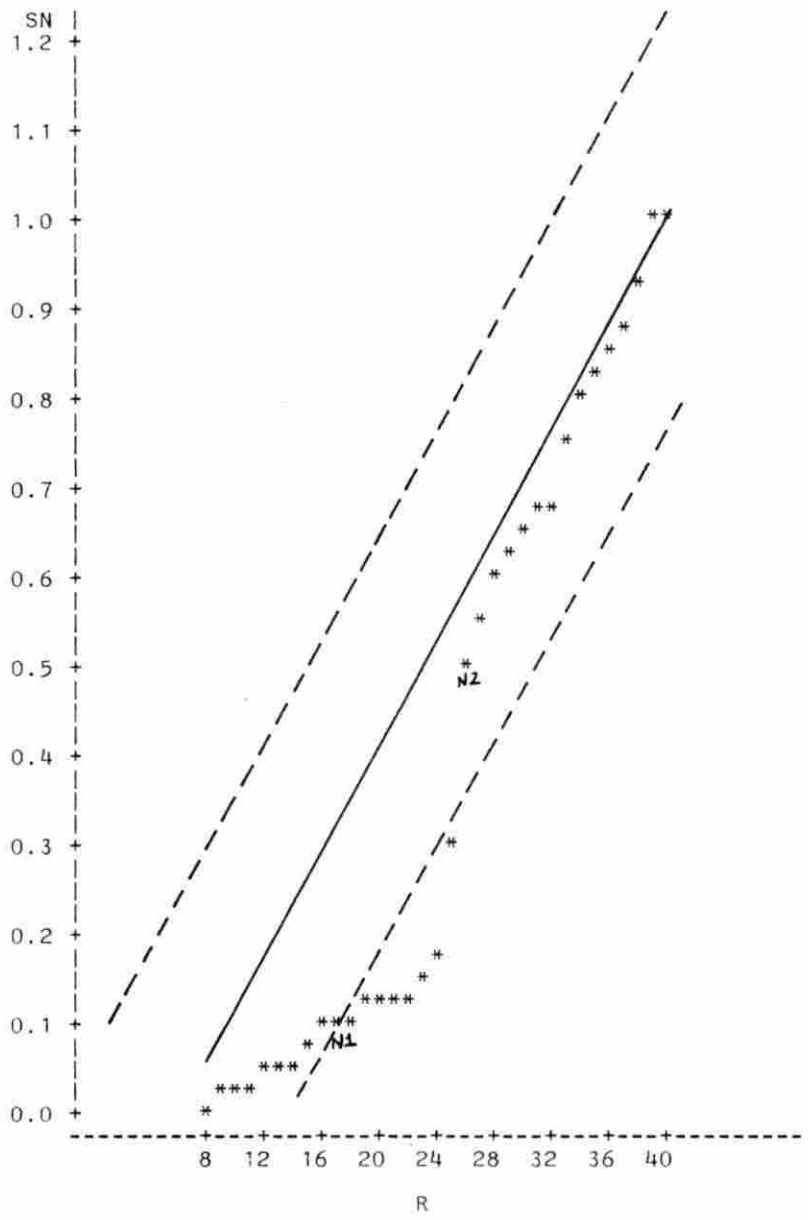

Figure 7. CUSUMSQ plot over time - nonfat dry milk

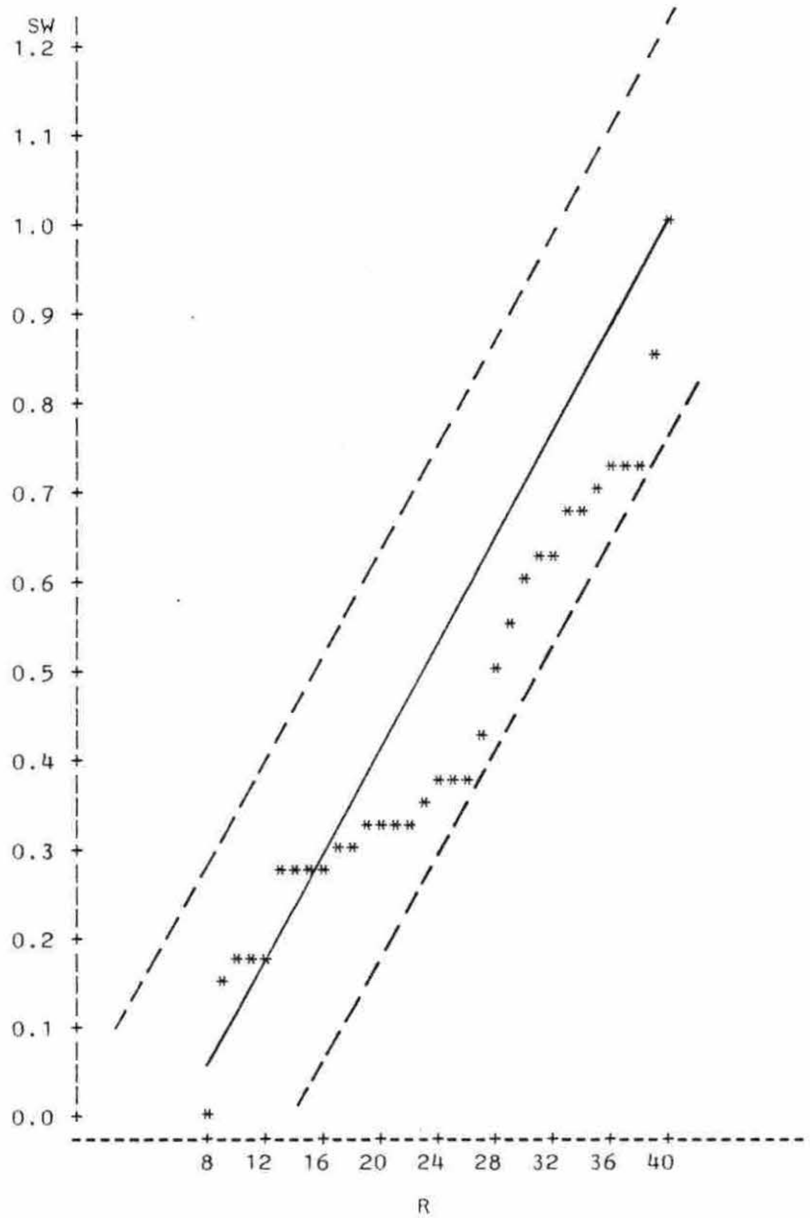

Figure 8. CUSUMSQ plot over time - dry whole milk

Table 1. CUSUMSQ (S_r) values over own price observation

OBS	R	SF	SE	SN	SW
1	8	0.00005	0.00409	0.00158	0.4609
2	9	0.00592	0.03740	0.01839	0.04851
3	10	0.00602	0.09132	0.01981	0.07041
4	11	0.00602	0.12587	0.05927	0.08546
5	12	0.06333	0.20978	0.06600	0.10295
6	13	0.07033	0.21311	0.07444	0.12043
7	14	0.07240	0.21506	0.07647	0.15553
8	15	0.11057	0.27303	0.10605	0.15555
9	16	0.12677	0.27622	0.10635	0.15710
10	17	0.18809	0.27898	0.10891	0.15747
11	18	0.21937	0.28504	0.13004	0.16865
12	19	0.22301	0.28843	0.13234	0.17417
13	20	0.24015	0.29638	0.13564	0.19385
14	21	0.26078	0.30172	0.13576	0.25697
15	22	0.37645	0.43409	0.14376	0.25764
16	23	0.38702	0.45177	0.17793	0.31340
17	24	0.39709	0.46468	0.37393	0.33341
18	25	0.49922	0.47034	0.40429	0.35734
19	26	0.51322	0.51043	0.41147	0.39291
20	27	0.52358	0.52182	0.41478	0.44926
21	28	0.57700	0.53136	0.41545	0.45829
22	29	0.58332	0.64740	0.44554	0.46055
23	30	0.59282	0.72048	0.45090	0.50703
24	31	0.60048	0.73009	0.45127	0.54654
25	32	0.71321	0.73028	0.48634	0.57667
26	33	0.71769	0.76576	0.48636	0.57669
27	34	0.80450	0.81434	0.51003	0.59550
28	35	0.80967	0.89992	0.68119	0.61123
29	36	0.81015	0.91363	0.94635	0.76126
30	37	0.87771	0.91802	0.99791	0.81119
31	38	0.91027	0.91992	0.99869	0.89713
32	39	0.98042	0.91994	0.99941	0.89726
33	40	1.00000	1.00000	1.00000	1.00000

Table 2. CUSUMSQ (S_r) values over time

OBS	R	SF	SE	SN	SW
1	8	0.01821	0.08328	0.01198	0.00280
2	9	0.11789	0.09742	0.01278	0.15665
3	10	0.18229	0.09756	0.01456	0.16753
4	11	0.19288	0.16684	0.02740	0.17149
5	12	0.19477	0.32147	0.04572	0.18420
6	13	0.21279	0.50467	0.04868	0.27079
7	14	0.21333	0.54117	0.04901	0.27950
8	15	0.21434	0.57815	0.08731	0.28010
9	16	0.22098	0.58116	0.09593	0.28029
10	17	0.23836	0.59238	0.10214	0.28771
11	18	0.23841	0.60302	0.10748	0.29988
12	19	0.27245	0.66431	0.11410	0.31793
13	20	0.33173	0.68169	0.12334	0.31941
14	21	0.37543	0.71880	0.12395	0.32846
15	22	0.48200	0.73280	0.12447	0.32854
16	23	0.52275	0.73940	0.14465	0.34735
17	24	0.56325	0.76819	0.17670	0.36535
18	25	0.56741	0.79255	0.29848	0.36826
19	26	0.59099	0.84761	0.49740	0.38057
20	27	0.59859	0.84787	0.54188	0.41275
21	28	0.63579	0.85122	0.61216	0.50030
22	29	0.69768	0.85569	0.62740	0.56238
23	30	0.73774	0.86133	0.63873	0.60110
24	31	0.82890	0.87199	0.67287	0.61524
25	32	0.84963	0.87719	0.68235	0.63238
26	33	0.85009	0.89180	0.76222	0.67473
27	34	0.85719	0.91566	0.81154	0.68286
28	35	0.85719	0.93945	0.81926	0.70901
29	36	0.87253	0.96472	0.86198	0.71561
30	37	0.88837	0.98209	0.87486	0.73592
31	38	0.90410	0.98227	0.91849	0.73610
32	39	0.94743	0.99543	0.98872	0.83848
33	40	1.00000	1.00000	1.00000	1.00000

8, respectively.

In Figures 1, 2, and 4, the computed values of cumulative sums of squares plotted against own price observations for fluid milk, evaporated milk, and dry whole milk, respectively, are all inside the 10% significance line. These plots provide the evidence that there is no break point over own price variables for these dairy products.

In Figure 3, the values of cumulative sums of squares are plotted against own price variable for nonfat dry milk. The intersections of the sample path and the significance lines N_1, N_2, N_3, and N_4 denote the possible points of coefficient change.

Figure 5 shows that when plotted over time, the cumulative sums of squares for fluid milk are all inside the 10% significance line. This plot provides evidence that the demand for fluid milk does not display structural change over the sample period. Therefore, the null hypothesis that there is no structural change in demand for fluid milk is not rejected.

In Figure 6, the values of cumulative sums of squares for evaporated milk are plotted against time. In this plot, E1 and E2 denote the intersections between the sample path and the significance lines. Therefore, it appears that 1951-52 and 1966-67 are points of structural change in the demand for evaporated milk.

In Figure 7, N1 and N2 are two points at which the sample path of cumulative sums of squares for nonfat dry milk cross the significance line. Thus, it appears that 1955-56 and 1964-65 are the points of structural change for nonfat dry milk.

Figure 8 plots the sample path over time of the cumulative sum of squares residuals for dry whole milk. The results obtained from this CUSUMSQ of recursive residuals graph are similar to the results presented in Figure 5 in that all the points of (r, S_r) lie inside the significance line. Therefore, it appears that the demand relationship for dry whole milk was stable over time.

Results of the Linear Spline Equation

The results of the previous section indicate that the demands for fluid milk, evaporated milk, and dry whole milk do not display coefficient changes. Therefore, we will apply the linear spline transformation to the demand for nonfat dry milk.

Nonfat dry milk

In the plot of CUSUMSQ recursive residuals for nonfat dry milk, N_1, N_2, N_3, and N_4 are four break points where the coefficients of the demand function may change. The corresponding values of the independent variables at N_1, N_2, N_3, and N_4 are presented in Table 3. These points are the break points of the linear spline transformation applied to nonfat dry milk demand model. Following the transformation procedure discussed in the previous chapter and repeating the procedure for all applicable independent variables, a linear spline demand function for nonfat dry milk can be expressed. The independent variables of this transformed equation are denoted as:

PFI: real price of fluid milk,

PFI2: the values of PFI greater than 0.0509178,

Table 3. Break point values for linear spline equation in nonfat dry milk

Points	Independent variable				
	PNI	PFI	PEI	PWI	YI
N_1	0.191397	0.0509178	0.0379888	0.385954	20.5064
N_2	0.214026	0.0534608	0.0401639	0.399089	28.5064
N_3	0.260974	0.0552712	0.0412718	0.458700	30.6074
N_4	0.363408	0.0561097	0.0471853	0.481546	33.6285

PFI3: the values of PFI greater than 0.0534608,

PFI4: the values of PFI greater than 0.0552712,

PFI5: the values of PFI greater than 0.0561097,

PEI: real price of evaporated milk,

PEI2: the values of PEI greater than 0.0379888,

PEI3: the values of PEI greater than 0.0401639,

PEI4: the values of PEI greater than 0.0412718,

PEI5: the values of PEI greater than 0.0471853,

PNI: real price of nonfat dry milk,

PNI2: the values of PNI greater than 0.191397,

PNI3: the values of PNI greater than 0.214026,

PNI4: the values of PNI greater than 0.260974,

PNI5: the values of PNI greater than 0.363408,

PWI: real price of PWI,

PWI2: the values of PWI greater than 0.385954,

PWI3: the values of PWI greater than 0.399089,

PWI4: the values of PWI greater than 0.458700,

PWI5: the values of PWI greater than 0.481546,

YI: real per capita income,

YI2: the values of YI greater than 20.7731,

YI3: the values of YI greater than 28.5064,

YI4: the values of YI greater than 30.6074, and

YI5: the values of YI greater than 33.6285.

Table 4 contains the transformed variables, the estimated coefficients, and the corresponding t-ratios from the results of the

Table 4. Estimated coefficients of linear spline demand equation in nonfat dry milk

	Coefficient	t-value
Constant	-10440	-0.772
PNI	-9223	-1.682
PNI2	19202	1.090
PNI3	-18293	-0.912
PNI4	28103	0.493
PNI5	-73693	-0.439
PFI	-44311	-0.101
PFI2	-86555	-0.193
PFI3	69109	0.439
PFI4	-171273	-0.211
PFI5	110878	0.108
PEI	150106	0.335
PEI2	64353	0.184
PEI3	-105057	-0.227
PEI4	-172654	-0.347
PEI5	0	0
PWI	3225	0.168
PWI2	-15998	-0.268
PWI3	14205	0.348
PWI4	-6768	-0.196
PWI5	32288	0.442
YI	417	1.019
YI2	-442	-1.182
YI3	128	0.458
YI4	-936	-0.409
YI5	0	0
R^2	0.9980	---

regression analysis. These results show a very high R^2 and all of the coefficients are not significantly different from zero at the 10 percent significance level. These results provide evidence of no slope changes in the nonfat dry milk demand equation for all variables. Thus, the null hypothesis that no discrepancy among coefficients of an independent variable for the spline demand equation is not rejected. In addition, the coefficients on PEI5 and YI5 are exactly zero since the number of observations for the fifth interval of variables PEI and YI is only one. This occurs since the value of break point for the variables PEI and YI is the biggest of all values of PEI and YI. Therefore, the slope of the spline over the fifth interval is no different from the slope of the fourth segment. The coefficients on the variables PEI5 and YI5 represent the change in the slope from interval 4 to interval 5 and is zero.

Results of the Chow Test

So far, attention has focused on the changes in the individual coefficients. It is possible that the effect of an individual coefficient change may be offset or reinforced by changes in the coefficients of the other independent variables. It is also possible that the demand equation may be unstable in the effects of other exogenous factors. Therefore, we consider the Chow test to examine the stability of the overall regression rather than the stability of individual coefficients.

According to the description in Chapter III, the Chow test needs _a priori_ information about the points of instability in order to test for structural change. The plots of the CUSUMSQ test shown in Figures 5 through 8 indicate that the points E1, E2 and N1, N2 divide the overall sample period into three intervals for evaporated milk and the nonfat dry milk, respectively, while also indicating stability of the demand for fluid and dry whole milk. The Chow test is a statistical method to test for the equality of two linear regressions. In this section, we first test the stability of the evaporated milk and nonfat dry milk demand relationships between the first and second periods, then examine the stability of these relationships between the second and third time periods.

Demand for evaporated milk

The points E1 and E2 divide the sample period into three segments: 1940-1951, 1952-1966, and 1967-1979. First, we examine whether the regression from the 1940-1951 period is different from the regression from the 1952-1966 period. We then examine whether the two sets of observations from the 1952-1966 period and the 1967-1979 period can be regarded as belonging to the same regression model.

The results of the regression analyses for the separate time periods are presented in Table 5. The estimated own price coefficients have the expected signs and significant t-ratios. All of the regression equations have the high R^2's except for the 1940-1951 period. The estimated income coefficients have negative signs. A reasonable

Table 5. Relation between per capita quantity consumption of evaporated milk and independent variables[a]

Independent variables	Regression equations					
	1940-1951	1952-1966	1967-1979	1940-1966	1952-1979	1940-1979
Intercept	17.13 (1.85)	17.42 (3.71)	12.23 (1.77)	24.73 (7.04)	21.81 (9.01)	24.87 (4.57)
PFI	154.02 (1.02)	336.11 (1.84)	305.23 (2.34)	147.13 (1.84)	164.29 (2.29)	142.62 (3.27)
PEI	-12.84 (-0.10)	-305.08 (-2.56)	-270.51 (-3.17)	-79.81 (-1.82)	-204.12 (-3.54)	-84.58 (-2.02)
PNI	-5.73 (-0.32)	14.94 (1.06)	-6.67 (-1.38)	-1.63 (-0.16)	-3.15 (-1.06)	-1.75 (-0.49)
PWI	-5.38 (-0.40)	8.46 (0.72)	14.91 (1.01)	7.96 (1.48)	18.90 (3.47)	8.64 (3.30)
YI	-0.16 (-0.33)	-0.71 (-5.85)	-0.50 (-2.81)	-0.84 (-10.84)	-0.74 (-13.43)	-0.84 (-17.55)
R^2	0.5996	0.9841	0.9844	0.9505	0.9947	0.9877
DF	6	9	7	21	22	34
DW	-	1.806	-	2.402	1.540	2.275

[a]Student-values are given in parentheses.

conclusion based on these results is that evaporated milk is an inferior good. A Durbin-Watson (DW) statistic is inappropriate for sample sizes smaller than fifteen; it was computed for only four of the estimated equations. The DW statistic for the 1952-1979 period (1.540) was within the inconclusive range of 0.83 and 1.62. The other three DW statistics indicated that the null hypothesis of no autocorrelation was not rejected.

The error sum of squares and the computed F statistics for the Chow test are shown in Table 6. The values of F obtained by comparing the first 12 observations with the second 15 observations is 1.265 with 6 and 15 degrees of freedom. The corresponding tabled F with a 5 percent significance level is 2.79. Thus, the computed value of F is not significant at the 5 percent level and we do not reject the hypothesis that the relationship is stable. The value of F computed by comparing the middle 15 observations with the last 13 observations is 1.36 with 6 and 16 degrees of freedom. Since 1.365 is smaller than the critical F value obtained from the F tables with a 5 percent significance level, we do not reject the null hypothesis that no structural change occurred over these two periods. Therefore, the data indicates that no change in the structure of demand for evaporated milk occurred during the 1940-1979 time period.

Demand for nonfat dry milk

The analysis of the nonfat dry milk data using the CUSUMSQ test procedure resulted in two break points, N1 and N2, which divided the

Table 6. Computed F value of Chow test for evaporated milk

Period I: 1940-1951
Period II: 1952-1966
Period III: 1967-1979

$ESS_{I} = 7.13059 \qquad T_1 = 12$

$ESS_{II} = 1.14141 \qquad T_2 = 15$

$S_U = ESS_I + ESS_{II} = 8.54472 \qquad d.f._U = 27-12 = 15$

$S_R = ESS_{I,II} = 12.86853 \qquad d.f._R = 27-6 = 21$

$$F_{III} = \frac{(S_R - S_U)/d.f._R - d.f._U}{S_U/d.f._U}$$

$$= F\binom{6}{15} = 1.265 < F_c\binom{6}{15} = 2.79$$

$ESS_{II} = 1.14141 \qquad T_2 = 15$

$ESS_{III} = 0.54058 \qquad T_3 = 13$

$S_U = ESS_{II} + ESS_{III} = 1.95471 \qquad d.f._U = 28-12 = 16$

$S_R = ESS_{II,III} = 2.95527 \qquad d.f._R = 28-6 = 22$

$$F_{II,III} = \frac{(S_R - S_U)/d.f._R - d.f._U}{S_U/d.f._U}$$

$$= F\binom{6}{16} = 1.365 < F_c\binom{6}{16} = 2.74$$

total sample period into three segments, 1940-1955, 1956-1964, and 1965-1979. Similar to the preceding discussion, we first test for the stability between the two regression equations for time periods 1940-1955 and 1956-1964, then for time periods 1956-1964 and 1965-1979.

Six demand equations for nonfat dry milk were estimated for the different time periods. The results of the regression analyses are given in Table 7. In general, the R^2's are high, and the estimated own price and income coefficients have the expected signs. The DW statistic was also computed for each of the equations estimated, except for the 1956-1964 equation. The DW values of 2.263 for the 1940-1955 period, 2.652 for the 1965-1979 period, 1.953 for the 1956-1979 period, and 1.757 for the 1940-1979 period resulted in the nonrejection of the null hypothesis of no positive or negative autocorrelation. For the equation for the 1940-1964 period, however, there is inconclusive evidence of serial correlation on the basis of computed DW statistics.

Following the procedure developed for the Chow test in the previous chapter, the F statistics are calculated under the two different null hypotheses and are presented in Table 8. The results of the F tests presented indicate that we reject the null hypothesis at the 5 percent significance level for the 1940-1955 and 1956-1964 periods, implying that there has been significant structural change in demand for nonfat dry milk during the 1940-1964 time period. The F test for the 1956-1964 and 1965-1979 time periods is not significant at 5 percent level. Thus, at the 5 percent significance level, there appears to be no structural change from period II (1956-1964) to

Table 7. Relation between per capita consumption of nonfat dry milk and independent variables[a]

Independent variables	Regression equations					
	1940-1955	1956-1964	1965-1979	1940-1964	1956-1979	1940-1979
Intercept	-1788.23	3837.17	-1001.07	-1126.30	601.84	634.22
PFI	26876.49	-41437.56	78551.20	26370.16	50559.22	15185.61
PEI	5012.86	60729.71	-50013.67	-4707.16	-24181.83	-19829.85
PNI	-227.53	-4774.62	-1814.40	-324.43	-1314.24	-2291.53
PWI	-3284.28	-2434.10	-1208.85	-2494.59	-3411.89	-636.72
YI	122.37	50.14	28.71	92.41	15.42	42.59
R^2	0.9216	0.9084	0.9079	0.9469	0.8050	0.8829
DF	10	3	9	19	18	34
DW	2.263	-	2.652	1.405	1.953	1.757

[a]Student-value are given in parentheses.

Table 8. Computed F value of Chow test for nonfat dry milk

Period I: 1940-1955
Period II: 1956-1964
Period III: 1965-1979

$ESS_I = 46811.58278$ $\quad T_1 = 16$

$ESS_{II} = 8010.34132$ $\quad T_2 = 9$

$S_U = ESS_I + ESS_{II} = 54821.92409$ $\quad d.f._U = 25-12 = 13$

$S_R = ESS_{I,II} = 132300.38604$ $\quad d.f._R = 25-6 = 19$

$$F_{I,II} = \frac{(S_R - S_U)/d.f._R - d.f._U}{S_U/d.f._U}$$

$$= F\binom{6}{13} = 3.062 > F_c\binom{6}{13} = 2.92$$

$ESS_{II} = 8010.34132$ $\quad T_2 = 9$

$ESS_{III} = 42109.30298$ $\quad T_3 = 15$

$S_U = ESS_{II} + ESS_{III} = 50119.64429$ $\quad d.f._U = 24-12 = 12$

$S_R = ESS_{II,III} = 113876.40400$ $\quad d.f._R = 24-6 = 18$

$$F_{II,III} = \frac{(S_R - S_U)/d.f._R - d.f._U}{S_U/d.f._U}$$

$$= F\binom{6}{12} = 2.544 < F_c\binom{6}{12} = 3.00$$

Table 8. Continued

$$S_U = ESS_I + ESS_{II,III} = 160687.9867 \qquad d.f._U = 40\text{-}12 = 28$$

$$S_R = ESS_{I,II,III} = 428391.366191 \qquad d.f._R = 40\text{-}6 = 34$$

$$F_{I,II,III} = \frac{(S_R - S_U)/d.f._R - d.f._U}{S_U/d.f._U}$$

$$= F\binom{6}{28} = 7.775 > F_c\binom{6}{28} = 2.45$$

period III (1965-1979). The regression equations for these different periods can be regarded as a single regression model. Hence, it is appropriate to present a demand equation which is estimated from the observations during the periods 1956-1979. Moreover, we would like to test whether the regression equation for period I (1940-1955) has the same form with the reestimated equation for periods II and III (1956-1979). The F value for this test is significant, and this result implies structural changes in the nonfat dry milk demand equations over the sample period.

Comparison of Coefficients Among Periods

The responsiveness of quantity demanded to prices and income, evidently have not been constant over the sample period for evaporated milk and nonfat dry milk. The elasticity coefficients from these demand relationships are shown in Table 9.

The direct own price elasticities of evaporated milk increased, and the demand was more price elastic toward the end of the sample period than at the beginning. This suggests that evaporated milk consumption has become more influenced by own prices in the last few years. In the case of nonfat dry milk, the demand was more price elastic in the middle than either end of the sample period.

As measured by the signs and magnitudes of cross elasticities, four basic changes in the demand interrelations are evident over the sample period. First, as exhibited by the cross elasticities of the evaporated milk price on nonfat dry milk consumption, there were

Table 9. Computed elasticities over the time intervals

	Year	Items				
		Fluid milk	Evaporated milk	Nonfat dry milk	Dry whole milk	Income
	1940-1951	0.556	-0.030	-0.062	-0.193	-0.126
Evaporated milk	1952-1966	2.097	-1.337	0.219	0.151	-1.473
	1967-1979	3.675	-2.340	-0.113	0.118	-2.255
	1940-1955	5.346	0.081	-0.242	-4.672	4.343
Nonfat dry milk	1956-1964	-2.527	2.357	-0.844	-0.870	1.224
	1965-1979	5.401	-2.605	-0.621	-0.860	1.431

structural shifts from a complementarity to a substitution relationship in the middle of the sample period. Second, the impact of the dry whole milk price on evaporated milk consumption shifts from a substitution to a complementarity relationship. Third, some of the shifts are unstable, as in the case of the impact of the nonfat dry milk price on evaporated milk consumption and the impact of the fluid milk price on nonfat dry milk consumption. Finally, there were nondirectional changes in the magnitudes of cross elasticities. The increase in the elasticity of evaporated milk consumption, with respect to the fluid milk price, indicates that evaporated milk consumption has become more influenced by fluid milk prices in the latter years. The income elasticities of demand for evaporated milk increased, but those of nonfat dry milk demand declined. Overall, the largest elasticity changes occurred in the demand for nonfat dry milk from period I to II.

CHAPTER V. ACCURACY OF CONSUMPTION FORECASTS

If the demand structure has changed, the estimated demand equation would be a hybrid applicable to neither the period before, nor after the structural change (Tomek and Robinson, 1972, p. 305). For consumption analysis purposes, accurate estimation and a clear understanding of past relationships between quantities and prices is important. Therefore, the evaluation of the predictive ability of a estimated equation can also support the hypothesis of structural change.

From the results of the previous chapter, it appears that the structural change has occurred in the demand for nonfat dry milk. The demand equations for different time periods (before and after the structural change and entire period) have been estimated. In this chapter, we will evaluate the forecasting accuracy of these different demand equations to provide other evidence of structural change.

A number of measures are commonly used to evaluate forecasting accuracy, such as Theil's U accuracy statistic (Madalla, 1977), mean-square error (Madalla, 1977), root-mean-square simulation error (Pindyck and Rubinfeld, 1981), and root-mean-square percent simulation error (Pindyck and Rubinfeld, 1981). In this section, we measure the accuracy of forecasts by the means of RMS error, RMS percent error, and Theil's inequality coefficient. First, we define these measures, and then we analyze the empirical results to evaluate the accuracy of consumption forecasts of the different demand equations for nonfat dry milk.

Root-mean-square Simulation Error

The RMS simulation error for the dependent variable CNt is defined as

$$\text{RMS error} = \frac{1}{T} \sum_{t=1}^{T} (\text{CNtp} - \text{CNta})^2$$

where CNtp = predicted value of CN,

CNta = actual value of CN, and

T = number of predicted values.

The RMS is thus a measure of the deviation of the predicted variable from its actual time path. The magnitude of this error could be evaluated relative to the average size of the variable in question.

Root-mean-square Percent Simulation Error

Another simulation error statistic is the RMS percent simulation error, which is defined as

$$\text{RMS percent error} = \frac{1}{T} \sum_{t=1}^{T} \left(\frac{\text{CNtp} - \text{CNta}}{\text{CNta}}\right)^2 .$$

This measures the deviation of the simulated variable from its mean in percentage terms.

Theil's Inequality Coefficient

This statistic is defined as

$$U = \frac{\frac{1}{T}\sum_{t=1}^{T}(CNtp - CNta)^2}{\frac{1}{T}\sum_{t=1}^{T}(CNtp)^2 + \frac{1}{T}\sum_{t=1}^{T}(CNta)^2}$$

where the numerator of U is the RMS simulation error, but the denominator is such that U will always fall between 0 and 1. If U=0, CNta = CNtp for all t, and the model has perfect predictive ability. If U=1, on the other hand, the regression equation has no forecasting ability.

Comparison of Consumption Forecasting Accuracy

Consumption predictions using nonfat dry milk demand equations for different time periods are shown in Table 10. The computed measures of forecasting accuracy, discussed in the previous section, are shown in Table 11.

From the results of CUSUMSQ test over time, we knew that 1955-56 and 1964-65 were two break points where structural change in the demand for nonfat dry milk might possibly have occurred. The Chow test provided additional evidence that structural change in the demand for nonfat dry milk occurred in 1955-56. Therefore, the estimated equation for the 1956-1979 period presents the demand equation for nonfat

Table 10. Actual and predicted per capita consumption for nonfat dry milk using different time period estimation equation (lb)

Year	Observed	Predicted			
		1940-1955	1956-1979	1965-1979	1940-1979
1980	688	2297	674.833	609	797.096
1981	612	2277	671.186	558	832.923

Table 11. Comparison of accuracy of consumption forecasts

Measure	Regression equations			
	1940-1955	1956-1979	1965-1979	1940-1979
RMS error	1637.24	42.87	67.66	174.23
RMS % error	2.537	0.0697	0.1024	0.4394
U	0.557	0.032	0.055	0.119

dry milk after the structural change, and the estimated equation for the 1940-1955 period estimates the demand equation before the structural change.

According to the results of Table 11, the values of the three different forecast accuracy measures for the demand equation estimated for the 1956-1979 period are the smallest. It appears that the demand equation estimated after the structural change has the greatest forecasting accuracy. This evidence supports the conclusion that a structural change occurred in 1955-56 for nonfat dry milk demand. The forecasting accuracy for the 1965-1979 estimated demand is greater than the forecasting accuracy for the 1956-1979 estimated demand. This result supports the hypothesis that no structural change occurred at the 1964-65 break point. This conclusion does not disagree with the result of the Chow test presented in the preceding chapter.

CHAPTER VI. SUMMARY AND CONCLUSIONS

The purpose of this study was to apply statistical techniques to empirically analyze structural change in the retail demands for milk products. The approach assumed that retail demand functions for fluid milk, evaporated milk, nonfat dry milk, and dry whole milk are linear functions. The conclusions reached in this study by means of the CUSUMSQ test indicate that break points in each commodity's own price cannot be found for all milk products in this study, except nonfat dry milk, and that the demand relationships are stable for fluid milk and dry whole milk over the entire time period of analysis. Moreover, the possible break points over price variable for nonfat dry milk and the possible points over time period at which structural change in the demand for evaporated milk and dry whole milk occurred were also estimated by means of the CUSUMSQ test. Based on the CUSUMSQ test, the results of the Chow test identified structural change to have occurred for nonfat dry milk in 1955-56, but structural change did not occur in the demand for evaporated milk. Evidence presented by the linear spline transformation supports the conclusion of no individual coefficient changes in the nonfat dry milk demand equations. Thus, independent variables are not the factors of the structural change. The structural change over all equations are the effects of the exogenous factors.

Own price, cross price and income elasticities for periods between the points of changes were computed. Within our analytical framework, it is evident that the demand interrelationships, and the price and income responses of evaporated milk and nonfat dry milk have changed considerably within the sample period. In particular, the empirical results suggest that the own price, cross price, and income elasticities of demand for nonfat dry milk have a great discrepancy between the 1940-1955 and 1956-1964 periods. This result does not disagree with the conclusions of the Chow test.

One final result from the evaluation of accuracy of consumption forecasts indicated that for nonfat dry milk demand, the regression equation estimated after the 1955-56 break point has the smallest RMS error, RMS percent error, and Theil's U values. The reasonable conclusions are that structural change in the demand for nonfat dry milk occurred in 1955-56, but not in 1964-65. The result of the Chow test is consistent with this conclusion.

Although the discussion in this paper has been limited to a linear demand model, we can extend the investigation for structural change to other kinds of functional forms. The demonstration of statistical analysis for structural change in this study was illustrated with the four specified milk products. For further extensions of this research, we can apply the method which was developed in this study to other products. Moreover, an area of further research may be to find the relative factors which affect the behavioral relationships to explain the reasons for this structural change.

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